Original scientific paper
(accepted October 15, 2016)

MEASURING PATH DEPENDENCY

Peter Juhasz [1]
Kata Varadi
Agnes Vidovics-Dancs
Janos Szaz

Abstract
While risk management gained popularity during the last decades even some of the basic risk types are still far out of focus. One of these is path dependency that refers to the uncertainty of how we reach a certain level of total performance over time. While decision makers are careful in accessing how their position will look like the end of certain periods, little attention is given how they will get there through the period. The uncertainty of how a process will develop across a shorter period of time is often "eliminated" by simply choosing a longer planning time interval, what makes path dependency is one of the most often overlooked business risk types. After reviewing the origin of the problem we propose and compare seven risk measures to access path. Traditional risk measures like standard deviation of sub period cash flows fail to capture this risk type. We conclude that in most cases considering the distribution of the expected cash flow effect caused by the path dependency may offer the best method, but we may need to use several measures at the same time to include all the optimisation limits of the given firm.

Keywords: risk management, project, firm, financial planning.

Jel Classification: G31; G32

INTRODUCTION

The term of path dependency refers to the case where current situation and behaviour of actors can only be understood when analysing historical events, in other words besides of where we are it is also important how we got there. But his term should not only be used in research of organisations (e.g. Zhu et al. 2006), theories of firm (Stack and Gartland 2003) or macro situation of countries (e.g. Hassink 2005 and Facchini 2013) but also in case of projects.

[1] **Peter Juhasz**, PhD, Associate Professor; **Kata Varadi**, PhD, Associate Professor; **Agnes Vidovics-Dancs**, PhD, Associate Professor; **Janos Szaz**, PhD, Full Professor Corvinus University of Budapest, Department of Finance, Budapest, Hungary.

For example, Richards and Bradley (2014) use this term to describe the special characteristics of university patents as a kind of real options. After their definition "*path-dependency means that the price of the patent at any given time depends not only on its value at expiry, but 'how it got there', or the path of its underlying value over time*". (Richards and Bradley 2014, 819) This means we may have not only complex financial products but also common projects (e.g. R&D) with path-dependency. The authors emphasise that in case of such projects we can not rely on traditional option risk metrics like "Greeks".

Johnson (2007) applies a similar concept for innovation in general. He underlines that after the start of a project, the risk coming from the range of possible paths (in focus of this paper) decreases as time goes by, but at the same time the risk of management decisions becoming path dependent (that means being determined by previous steps – key in organisation research) increases, and phenomena like over-commitment may emerge.

It is easy to see that even the simplest projects may have path-dependent traits. When predicting long term cash flows, we need to split the total lifetime of the projects into shorter periods so we may calculate time values of payments. But the length of these shorter time intervals used is often chosen to maximise the precision of prediction of financial statements. Later, when estimating the cost of capital the risk is often linked to the standard deviation of the so predicted (less volatile) results only.

It is very common to overlook the importance of the choice about the length of these shorter periods. For example arriving to a certain level of yearly sales and produced quantity does not mean earning the same return at the same risk no matter how the production and sales are distributed across time within that predicted year. An even distribution of both sales and manufactured quantities implies less capital need, far lower risk of bankruptcy and higher profit than what we would realise in case of huge fluctuations even if the total volume will add up to the same amount. The uncertainty about how the business performance will reach the predicted level is often dropped and forgotten about at later stages of valuation.

Building on earlier simulation results (Juhasz, Szaz, and Vidovics-Dancs 2014, Vidovics-Dancs, Juhasz and Szaz 2014) our paper focuses on inventing and comparing possible risk metrics for path dependency in case of projects or firms. In addition to the standard deviation reflecting the uncertainty about where we will get to these new measures should also be used to reflect the uncertainty about how will get there.

1. MEASURING PATH DEPENDENCY IN THE LITERATURE OF FINANCE

The term path dependency used in economics and social sciences, evolved from physics, mathematics and biology. Facchini (2013) underlines that in social sciences there are two types of institutional path dependencies: the dependency on the past practice and experience, and dependency on imaginable possible future states of world. These together may very strongly lock in a certain kind of institutional functioning.

Path dependency is important in economics, since the results of the equilibrium models that do not take into account the way the process is getting to the equilibrium state, could be quite questionable. After the Encyclopedia of Law and Economics by Liebowitz and Margolis path dependency has three forms: "*weak (the efficiency of the chosen path is tied with some alternatives), semi-strong (the chosen path is not the best but not worth fixing), or strong (the chosen path is highly inefficient, but we are unable to correct it)*".

In finance this term is also used, mainly on the field of pricing derivatives, e.g. Asian or barrier options like bonus certificates (e.g. Shreve 2005, Haug 2007, Neftci 2008, Baule and Tallau 2011, Wusterhausen 2015), or in risk management (Domotor 2012). In this paper we will use this term the same way as Juhasz, Szaz and Vidovics (2014) did, namely that the final payoff (of a project or an asset) is influenced by past events which are out of our control, so we cannot effect the events by our own decisions.

In finance path dependency also is used in weak and strong form as in economics in general (Wilmott 2006). When we talk about the weak form of path dependency in derivative pricing, we mean that the price of the derivative depends only on the price of the underlying asset and time, like in case of a barrier option, while if we talk about the strong form, it means that we need another variable as well, like in case of an Asian option (for more details see Wilmott 2006).

Although the financial literature of path dependency is widespread, there is no research on how to quantify risk related to path dependency. In case of derivatives, focus is on how to price an option in case of path dependency. In the case of risk management, the key question is how to determine the optimal hedge ratio once our position is path dependent.

Haug (2007) gave a solution to measure differences among possible paths by calculating the volatilities of each of the paths. But the author aimed not really quantify path dependency, rather the goal was to show how the change in volatility in discrete models effect the delta hedging in case of options.

Still, paths appear in risk management but in different contexts. For example Qazi at al. (2016) and Eybpoosh, Dikmen and Birgonul (2011) use the expression risk path to describe connections and causalities among risk factors. Risk management uses critical path for the most time consuming process path the task could be performed (Zammori, Braglia and Frosolini 2009), while the term possible risk path refers to the riskiest of those.

2. PROBLEM DESCRIPTION

When predicting the result of a process where we see some volatility around the expected value for one single period, merging several periods and estimating the sum of those realisations can be far more exact. This is particularly true, once the extent of possible fluctuations around the expected value is far smaller in than the expected value itself.

For example, when rolling a dice the expected value is 3.5 while standard deviation is 1.7. But if we roll 10 dices at the same time and predict the sum of numbers, the expected value is 35 with a standard deviation of 5.4, in other words relative standard deviation (std dev/average) has decreased from 0.49 to 0.15. So we are more exact to predict sums than individual outcomes.

This phenomenon appears in corporate finance when predicting financial performance and selecting the time-breakdown of the plan. Very often managers see it impossible to have a realistic daily, weekly, or monthly prediction so they use quarters, half years, or for long term planning even years as planning intervals. Than when measuring the risk behind the predicted performance usually the standard deviation is used as measurement tool. We even see in a lot of finance books that the risk of a share investment should be measured by the standard deviation of its monthly, quarterly, or yearly return. This assumes that it is only where we get to, what counts, but how we got there is unimportant.

It is easy to show that in many cases paying only attention to the final result (e.g. yearly sum of daily cash flows) is oversimplifying the risk measurement problem. For the shake of this paper we will work with four different projects that only last for a year and all of them produce a yearly cash flow of 12. (Table 1) In the classic view this would mean they have the same risk, and some would even say those are riskless as the final cash is granted.

Table 1. Monthly cash flows of the four sample projects

Project	1	2	3	4	5	6	7	8	9	10	11	12	Sum
A	1	1	1	1	1	1	1	1	1	1	1	1	12
B	0	0	0	0	0	0	0	0	0	0	6	6	12
C	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3	12
D	-6	0	0	0	0	0	0	0	0	0	6	12	12

Of course, if we were sure to realise exactly those cash flows in Table 1, we could make a monthly breakdown and make our model more precise. But the cash flows are not ordered across time, rather in increasing order. We know, that the firm would realise these monthly cash flows but the order of realisation is unclear. This is the only uncertainty linked to the projects.

In real life of course there would be also some fluctuation around the expected values. The problem is, that in the classic methodology that joint standard deviation would be the only risk to be linked to these projects, just like in case of rolling dices.

Why does the order of cash flows matter? Assume the company has some starting cash balance. If negative cash flows happen earlier we may go bankrupt before realising the remainder of the inflow. Also if we may earn some return on positive cash balance receiving positive inflows earlier may produce a higher profit by the end of the year compared to getting those only later during the year. In reality, we may face such path dependency at several levels across the operation. The order of appearance of fluctuating demand, production capacity, sales collection, paying to suppliers, interest and tax rate changes, all could play a key role in the shareholder value creation of the given company. For our previous modelling results see Juhasz, Szaz and Vidovics-Dancs (2014).

3. ACCESSING PATH DEPENDENCY

(Relative) Standard deviation. Once the problem is understood, the first idea might be to calculate the standard deviation of the expected monthly values. (Table 2) Still, there are at least two good reasons why we would not recommend to build your investment decision on those numbers. (1) Project B with positive cash flows only seems to be more risky than project C where some bankruptcy risk may emerge. (2) If we would have some information on the order of cash flows, we could not integrate that into this measure, as it is exactly the order of elements that would not influence this risk measure. In our example relative standard deviation and standard deviation would be the same measure as expected values for the projects are the same.

Relative range. Once we calculate the difference between the highest positive and highest negative possible monthly change, we get the maximum range of change. Comparing that to the standard deviation provides lower values for a distribution with less frequent extreme values. Once some extreme values generate most of the standard deviation the order of appearance of possible cash flows becomes more important.

Table 2. Standard deviation and relative range of the monthly cash flows

Project	Std. dev.	Rel. std. dev.	Min	Max	Range	Relative range
A	0.00	0.00%	1.00	1.00	0.00	0.00
B	2.34	19.46%	0.00	6.00	6.00	2.57
C	2.09	17.41%	-1.00	3.00	4.00	1.91
D	4.31	35.89%	-6.00	12.00	18.00	4.18

To measure the risk coming from the order of the cash flows, adding them step-by-step may help to better understand the process. Figure 1 illustrates how the sum of cash flows could develop form period to period.

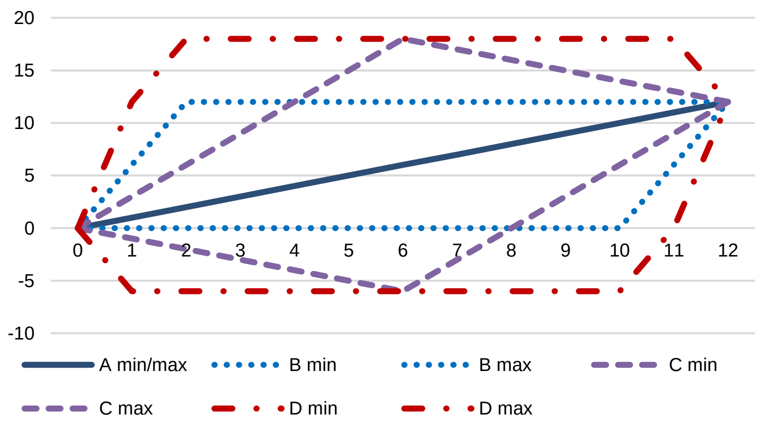

Figure 1. Possible paths of accumulating cash flows

As Figure 1 shows project C maximum and minimum levels are further away from the zero risk project A than those of project B that may indicate higher risk. It is also clear to see that contrary to project C the sum of cash flows can never be negative for project B.

Probability of bankruptcy. Of course extremities, or even negative values are not certain to happen, but those the probabilities are not shown on Figure 1. So a potential risk measure could be the likelihood of going bankrupt. Note that this may only be quantified for a given level of starting cash amount. To estimate probabilities in this paper we used Monte Carlo simulation a technique that is very common in valuing path dependent options (Wusterhausen, 2015) and project risk management (Cagliano, Grimaldi, and Rafele 2015). For each analysis 10000 random realisations were created. (Note that there are 1, 66, 924, and 1320 possible orders of CF for A, B, C, and D projects respectively, so all outcomes are very likely to be covered.) Table 3 indicates bankruptcy probabilities for some starting cash levels.

Table 3. Probability of bankruptcy

Cash at start	A	B	C	D
0	0.00%	0.00%	52.55%	27.48%
1	0.00%	0.00%	24.87%	27.48%
2	0.00%	0.00%	6.77%	27.48%
3	0.00%	0.00%	1.51%	27.48%
4	0.00%	0.00%	0.29%	27.48%
5	0.00%	0.00%	0.29%	27.48%
6 and above	0.00%	0.00%	0.00%	0.00%

This risk measure could be generalised by using stochastic calculus. Once the initial position and the sum of the cash flows is known the problem is similar to determining the probability of a Brownian bridge touching a predefined barrier (in this case the zero level). That technique is presented in detail for a similar problem by Bedini, Buckdahn, and Engelbert (2016).

Capital need to evade bankruptcy. Once we see the probability of bankruptcy at a certain starting cash level, we may compare how much more capital would be needed to evade bankruptcy. For to decrease probability of bankruptcy to zero, we need to consider the minimum points on Figure 1. So for A and B this value is zero, for C and D it is 6. As we see, based on this measure most of the risk differences remains hidden. Also, the annual cost of covering capital need could be more relevant for a business entity and that could be considered as an additional cash flow.

Expected cost of evading bankruptcy. In case of this measure we combine the probability of bankruptcy with the cost of capital emerging to hinder the cash shortfall. For this measure the cost of capital needs to be quantified (in our example that is set to 1.0 percent for a month). Also we need to consider how easy it is for the company to get fresh capital. If it is very hard, and they can not flexibly raise additional financing, all the capital needs to be provided right at the start of the project, and the cost of capital will emerge through all the year, what will lead us back to the preciously considered Capital need to evade bankruptcy measure.

If there is a monthly cost of financing we may quantify the expected amount of additional expenses, and also the whole distribution or at least the standard deviation or vale at risk (V@R) of that. Our results are shown on Figure 2.

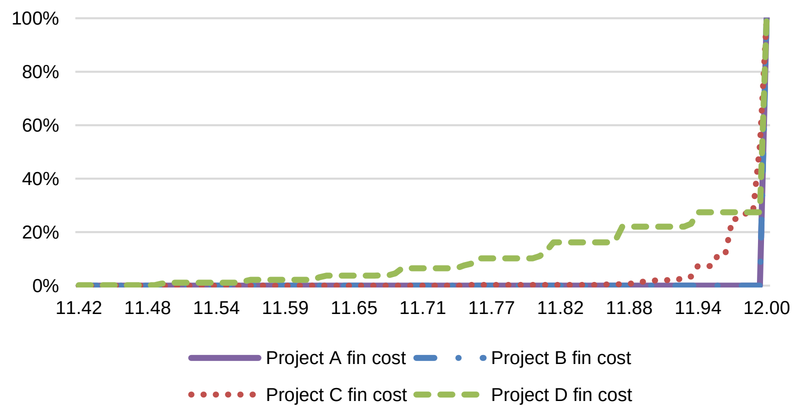

Figure 2. Cumulative distribution of total cash flow with financing costs

As Figure 2 shows, the distributions are asymmetric, so symmetric risk measures as standard deviation should be used very carefully when describing the possible outcomes. Expected loss (shortfall) or value at risk could be a better alternative.

Financial effect of path dependency. Our final measure does not only include all ideas presented until now, but also adds that there might be financial income collected on positive cash balance generated over the whole year (in our model 0.5 percent per month was used). This method is very similar to combining Monte Carlo analysis with Expected Monetary Value technique used in project risk management (Cagliano, Grimaldi and Rafele 2015). The measure should assume a starting cash amount of zero, so any profit on the free cash available at the start of the year included in the yearly plan may remain unaffected. (Figure 3)

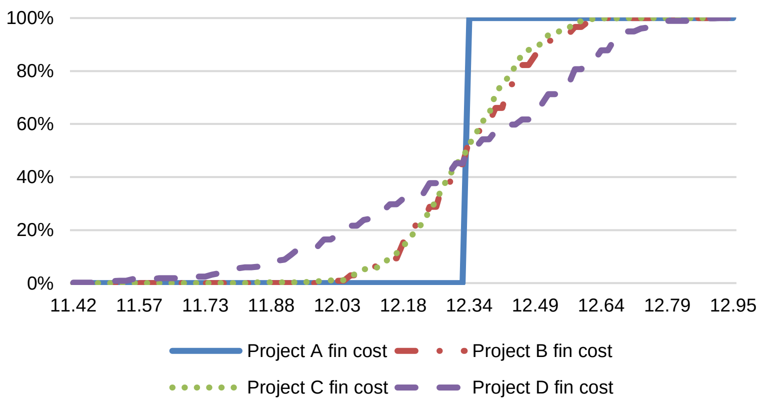

Figure 3. Cumulative distribution of total cash flow with total financial effect

Still, we have to be very careful, as in some cases when predicting the performance of the whole year the last year financial income due to path dependency is also included in the plan. In that case, as a first step we have to remove the value included, as unfortunately, that number may be a very poor estimate on the expected value as that is based on one single or just a handful random realisation only. After cleaning the plan this way, we may compare our findings on the distribution of total financial effect with the expected yearly performance.

Table 4. Risk metrics for the projects

	A	B	C	D	Description
1. Standard deviation	0.0000	2.3355	2.0889	4.3064	Symmetric. Blurs rare extreme and common average fluctuations
2. Relative standard deviation	0,00%	19,46%	17,41%	35,89%	
3. Relative range	0.0000	2.5690	1.9149	4.1798	Symmetric. Punishes rare, but extreme fluctuations.
4. Prob. of bankruptcy (starting cash=0)	0.00%	0.00%	52.55%	27.48%	Asymmetric. Focuses only on probability of negative outcome.

Table 4. (*continued*)

	A	B	C	D	Description
4. Prob. of bankruptcy (starting cash=3)	0.00%	0.00%	1.51%	27.48%	
4. Prob. of bankruptcy (starting cash=5)	0.00%	0.00%	0.29%	27.48%	
5. Capital need to evade bankruptcy	0.0000	0.0000	6.0000	6.0000	Asymmetric. Total capital need without considering the time while it is needed.
6. Expected cost of evading bankruptcy	0.00%	0.00%	1.48%	5.54%	Asymmetric. Financing costs only.
6. Var 95% cost evading bankruptcy	0.00%	0.00%	6.31%	31.54%	Asymmetric. Highest financing costs only.
7. Expected total financing effect	33.56%	34.22%	33.02%	32.59%	Symmetric. Includes all effects.
7. Var 95% total financing effect	34.19%	9.78%	8.25%	-20.73%	Asymmetric. Includes all effects but shows worst cases only.

To provide an overview, our results are summarised in Table 4. Based on the results we have to conclude that path dependency should not be measured by the standard deviation of the possible monthly cash flows itself, rather we should focus on where those monthly changes (cash flows) could lead to (bankruptcy or extra financial income). Relative range may be a very good option to estimate the importance of the cash flow order, but being a symmetric measure downside and upside risk cannot be clearly identified by using that measure only.

While most of the measures lists project D as the riskiest probability of bankruptcy at a zero starting cash level underlines how important it is to focus on risk measures adequate to the given situation. There we clearly see that project C has more chance to default, though expected cost of evading that is less. This is why we believe that measuring path dependency should never be done by just applying one single measure, rather the expected distribution of costs and benefits under the given limits of the firm in question should be considered.

CONCLUSIONS

Our paper considered seven possible risk measures of path dependency for cash flows. For testing four fictive projects with different possible monthly cash flows have been used. For each of those a Monte Carlo simulation of 10 thousand realisations has been performed to identify final cash distributions.

Based on the results it can be concluded that instead of focusing on the size and heterogeneity of the monthly cash flows one should focus on the possible paths the summarised cash flow may get along. That is why classic simple risk metrics like standard deviation or relative range are of limited use in measuring path dependency. At the same time both well performing symmetric and asymmetric rick measures are available, but to get a good overview it is advisable to use several measures at the same time.

REFERENCES

Baule, Rainer, and Christian Tallau. 2011. The Pricing of Path-Dependent Structured Financial Retail Products: The Case of Bonus Certificates. *Journal of Derivatives* 18 (4): 54–71.

Bedini, Matteo Ludovico, Rainer Buckdahn, and Hans-Jurgen Engelbert. 2016. "Brownian Bridges on Random Intervals." *Teor. Veroyatnost. i Primenen.* 61 (1): 129 –157.

Cagliano, Anna Corinna, Sabrina Grimaldi, and Carlo Rafele. 2015. Choosing project risk management techniques: A theoretical framework. *Journal of Risk Research* 18 (2): 232–248.

Domotor, Barbara M. 2012. *Market risk hedging under liquidity constraints.* PhD diss. Corvinus University of Budapest. http://phd.lib.uni-corvinus.hu/815/2/Domotor_Barbara_den.pdf (accessed March 2, 2016).

Eybpoosh, Matineh, Irem Dikmen, and M Talat Birgonul. 2011. Identification of Risk Paths in International Construction Projects Using Structural Equation Modeling. *Journal of Construction Engineering and Management*-Asce 137 (12): 1164–1175.

Facchini, Francois. 2013. Economic Freedom in Muslim Countries: An Explanation Using the Theory of Institutional Path Dependency. *European Journal of Law and Economics* 36 (1): 139–167.

Hassink, Robert. 2005. How to unlock regional economies from path dependency? From learning region to learning cluster. *European Planning Studies* 13 (4): 521–535.

Haug, Espen G. 2007. *Derivatives: Models on models.* Chichester, UK: John Wiley & Sons.

Johnson, William H. A. 2007. Managing Uncertainty in Innovation: The Applicability of Both Real Options and Path Dependency Theory. *Creativity & Innovation Management* 16 (3): 274-281.

Juhasz, Peter, Janos Szaz, and Agnes Vidovics-Dancs. 2014. Path Dependency in Corporate Finance. In *Proceedings of the SSEM EuroConference 2014*: *The International Conference on Emerging Markets Business, Economics, and Finance, 6–8 July, 2014,* 100–120. Budapest: Budapest University of Technology and Economics.

Neftci, Salih N. 2008. *Principles of financial engineering*. Burlington, USA: Academic Press.

Qazi, Abroon, John Quigley, Alex Dickson, and Konstantinos Kirytopoulos. 2016. Project Complexity and Risk Management (ProCRiM): Towards modelling project complexity driven risk paths in construction projects. *International Journal of Project Management* 34 (7): 1183–1198.

Richards, Timothy J., and Bradley J. Rickard. 2014. Patents as options: path-dependency and patent value. *European Review of Agricultural Economics* 41 (5): 817–841.

Shreve, Steven 2005. *Stochastic Calculus for Finance I: The Binomial Asset Pricing Model*. New York, USA: Springer.

Stack, Martin, and Gartland Myles P. 2003. Path Creation, Path Dependency, and Alternative Theories of the Firm. *Journal of Economic Issues* 37 (2): 487–494.

Vidovics-Dancs, Agnes, Peter Juhasz, and Janos Szaz. 2014. Path Dependency in Investment Strategies: A Simulation Based Illustration. In *ECMS 2014 Proceedings*, ed. Flaminio Squazzoni, Fabio Baronio, Claudia Archetti, and Marco Castellani, 758–764. European Council for Modeling and Simulation. doi:10.7148/2014-0758

Wilmott, Paul. 2006. *Paul Wilmott on quantitative finance*. 2nd ed. Chichester, UK: John Wiley & Sons.

Wusterhausen, Frank. 2015. An Analysis of Path-Dependent Options. *Journal of Optimization Theory & Applications* 167 (3): 874–887.

Zammori, Francesco A, Marcello Braglia, and Marco Frosolini. 2009. A fuzzy multi-criteria approach for critical path definition. *International Journal of Project Management* 27 (3): 278–291.

Zhu, Kevin, Kenneth L. Kraemer, Vijay Gurbaxani, and Sean X. Xu. 2006. Migration to open-standard interorganizational systems: Network effects, switching costs, and path dependency. *Mis Quarterly* 30 (1): 515–539.